Sound propagation in urban areas: A periodic disposition of buildings

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A numerical simulation of background noise propagation is performed for a network of hexagonal buildings. The obtained results suggest that the prediction of background noise in urban spaces is possible by means of a modified diffusion equation using two parameters: the diffusion coefficient that expresses the spreading out of noise resulting from diffuse scattering and multiple reflections by buildings, and an attenuation term accounting for the wall absorption, atmospheric attenuation, and absorption by the open top. The dependence of the diffusion coefficient with geometrical shapes and the diffusive nature of the buildings are investigated in the case of a periodic disposition of hexagonal buildings. [S1063-651X(99)12809-5]

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I. INTRODUCTION

Noise propagation in urban areas is a problem of major importance for people living in modern large cities and has attracted considerable attention in recent years. The sound events occurring in the streets are diverse and multiple. Proximity sounds act individually and their influences on people depend greatly on psychoacoustic factors. Conversely, for long distances, noise is the result of a mixing of sound produced by many sources, and may be considered as a stationary background *color*. In contrast to the proximity sounds, the background noise does not propagate but rather spreads out because of multiple reflections and diffuse scattering on building facades, making direct propagation impossible. This paper is concerned mainly with this background noise propagation.

Because of its major importance, the noise propagation in urban spaces has been studied for a long time. Wiener, Malme, and Gogos [1] was among the first to propose an experimental and a theoretical study of sound propagation in urban areas for sources close to the floor. Considering a street as a long channel with partially absorbing walls and a perfectly reflecting bottom, the image source theory was used to predict the sound intensity along streets. Applying the method of virtual sources, Schlatter [2], Sergeev [3], and Vinokur [4], also gave expressions for the stationary sound level close to and far from the source. With a different approach, assuming that all propagating modes of the street carry equal sound power, Davies [5] proposed expressions for the sound attenuation in the streets. Although interesting, Lyon [6] showed great differences between these models based on specular reflections and experimental data. It was suggested that the scattering of sound and multiple reflections on building facades are very important and could account for these discrepancies. As shown by Fig. 1, the acoustic field in a street may indeed be split up into an elemental field (direct sound and early reflections) and a diffuse sound field steming from the multiple reflections and scattering of sound by the facade irregularities like building edges, balconies, building stones, etc. Donavan [7] then proposed a model of sound propagation accounting for surface scattering. In the same way, in order to take the effects of scattering from objects and protrusions into account, Bullen and Fricke studied the sound field in streets in terms of propagation modes and gave some expressions for sound attenuation in streets [8] and sound propagation around street corners [9]. Other papers relating to traffic noise-level prediction in streets need also to be mentioned [10,11]. In another paper, assuming that the scattering of sound produces a diffuse field, Davies [12] gave a relation for the sound attenuation along streets. This assumption of diffuse sound field propagating in streets, created by multiple diffuse reflections on the side walls is, twenty years later, still used and discussed [13–17].

Sound propagation in cities is a much more complicated problem because of the presence of many streets, intersections (Fig. 2), and buildings with various shapes (Fig. 3). The first models of sound propagation in urban spaces were carried out in the early 1970's with the study of Shaw and Olson [18]. Cities were regarded as a plane surface with many randomly distributed sources. The model of sound propagation takes into account the inverse square-law transmission, the atmospheric absorption, and a shielding factor expressing the presence of obstacles in the propagation path. With a similar approach, Davies and Lyon [19] proposed a cell model to describe the effects of multiple barriers on the noise propagation in urban spaces. More simply, Lee and Davies [20] provided a nomogram (an abacus) for the prediction of sound levels generated by a sound source in a

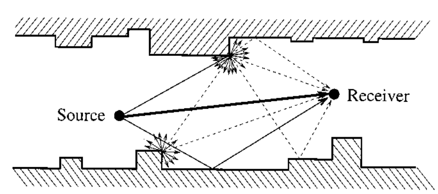

FIG. 1. Sound scattering and multiple reflections by irregular building facades on both sides of the street. The thick line, the full lines, and the broken lines, respectively, represent the direct, the reflected, and the diffuse-scattering fields.

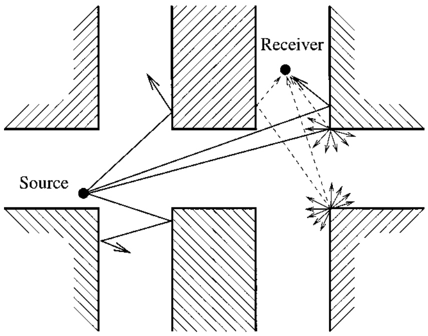

FIG. 2. Sound propagation in a street with intersections. The full lines and the broken lines, respectively, represent the reflected and the diffuse-scattering fields.

street, accounting for street width, distance from source, wall absorption, and street intersections. More recently, Makarewicz [21] proposed a mathematical theory of sound propagation within urban areas, taking into account propagation over a flat surface, specular reflection, and scattering by building facades. However, the complexity of the theory makes the model difficult for practical applications.

Because sound diffraction occurs in urban areas at house and street corners, balconies, and plays a major role Walerian [22] also proposed approximative solutions of multiple diffraction at edges and right-angle wedges [23]. Other paper using a geometrical approach by ray tracing and image sources were also investigated [24–28], but in view of the complex geometry of urban areas, such models are too difficult to use and need large computation times. Other methods have also been proposed to predict traffic noise in urban areas. However, most of them do not use models of sound propagation but simply a prediction of noise level from factors affecting noise propagation like buildings [29,30] and reflections by facades [31,32]. Last, recent works to predict traffic noise levels by way of neural networks have been proposed, and are currently in progress [33].

Thus, because an exact description of sound propagation in urban spaces is unrealistic, and because mathematical and physical models of sound propagation are often complicated and need important computations, some authors have turned toward statistical descriptions.

Considering an external reverberation as a limit case of reverberation in rooms, Yeow [34] was one of the first to propose a statistical model. In particular, an expression for the spatial distribution of the steady-state sound field was given and compared with experimental results. During the same year, Bullen [35] and Bullen and Fricke [36] proposed two statistical models based on the propagation of sound particles (called phonons) in a complex randomly distributed urban area described only by the mean-free-path and the average absorption coefficient. Experimental verifications showed that sufficient accuracy is found with little information about the environment. In a similar way, Leschnik [37] proposed to describe the sound propagation as a multiple-scattering process by considering houses and buildings as scattering objects. In view of good agreements with experimental data, and because this method is simple, Leschnik proposed to apply the model to the prediction of noise in urban spaces. Kuttruff [38] gave also an equivalent model, leading to the same conclusion. Finally, Walerian and Janczur [39] applied a model of noise propagation in industrial halls, based on a diffusion equation and derived by Kurze [40], to urban areas.

The point of view adopted in this paper is to consider the buildings as they are, in a deterministic way. In contrast, the reflections of the sound rays against the building facades are regarded probabilistically, because it is impossible first to take into account in detail the geometry of building facades and second, in order to simulate diffraction effects. In other terms, the scale of our study is a human scale, smaller than the buildings, but greater than the details of the facades. The advantage to consider noise propagation at this scale is obvious since the results can be applied to realistic cities. Nevertheless, the drawback is that it is necessary to consider the real arrangement of buildings. In this paper, the problem is avoided by the use of a periodic arrangement of buildings. One of the interests of this modeling is to show that the probabilistic treatment of the reflections is sufficient to produce a spread of noise. Further, it is possible to explore the dependence of this spreading out in terms of the geometrical dimensions of the streets, atmospheric attenuation, and wall absorption.

In Sec. II, a description of the periodic city used in the model is given. The use of a rather unusual hexagonal disposition for the buildings is discussed. Complete definitions of the reflection laws used in this paper are also given. It implies the use of a random law depending on the sound frequency and on the wall roughness. By way of illustration, the cases of both usual uniform and Lambert's law, and of two semidiffuse laws corresponding to different ranges of frequency, are analyzed in detail. A description of the simulation and applications are proposed in Sec. III. Then, a statistical study of the propagation of sound rays in the streets proves that they approximately obey a diffusion law, defined by a diffusion coefficient depending on street widths, frequency, and roughness of the building facades. It is important to note that the diffusion law is obtained as a consequence of the reflection laws and not from an additional hypothesis as often done in other papers. Finally, it is suggested in Sec. IV that a diffusion equation including extra attenuation terms can be used to investigate the noise propagation in a realistic built-up area, in place of ray statistics implying extensive computations.

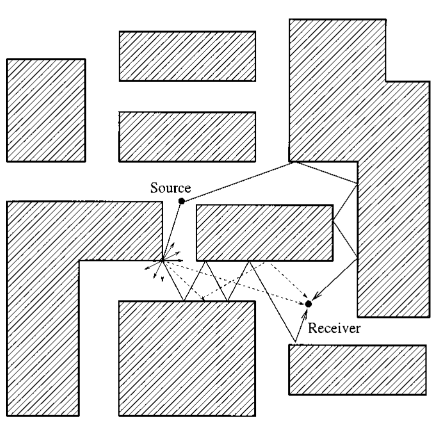

FIG. 3. Sound propagation in a typical urban space. The full lines and the broken lines, respectively, represent the reflected and the diffuse-scattering fields.

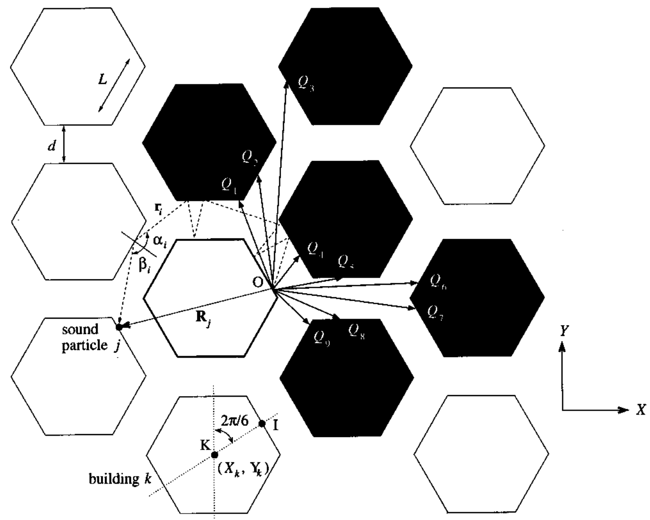

FIG. 4. Geometry of the picked urban space: periodic network of hexagonal buildings. d and L are, respectively, the street width and the length of a building facade. $Q_1, Q_2, \ldots, Q_9$ represent the 9 retained cases for a sound particle to propagate from a point source O on a half length of facade to another building (filled in black). In view of the symmetry in relation to the center (X_k, Y_k) of the building k by a rotation with an angle $2\pi/6$ and the symmetry in relation to the middle I on a facade, $9\times6\times2=108$ cases are considered on the whole. $\mathbf{R}_j$ and $\mathbf{r}_i$ are, respectively, the distance between the source and the sound particle j at time t, and an elemental displacement i of the particle. α_i and β_i are, respectively, the incident and the reflected angles (with the normal to the wall), corresponding to the collision i. The broken line represents the path of the sound particle in the network.

II. GEOMETRY OF THE URBAN SPACE

A. A network of hexagonal buildings

The present model is inspired by the so-called Lorentz model [41] used in solid-state physics, where a light particle collides with heavy particles at rest; in the present study the rays replace the trajectories. The choice for the definition of the proposed city model follows two ideas; (i) because the spatial distribution of buildings is periodic, it is possible to define a large urban space with only a few rules, and (ii) one wish to have a finite mean flight for the acoustic rays. A good way to verify this latter property is to avoid the possibility of a sound ray to propagate directly towards infinity without any collision: one says that the *horizon is finite*. By disposing parallel rectangular buildings, it is not so easy to have both a periodic distribution of buildings and a finite horizon. The solution of a network of hexagonal buildings shown by Fig. 4 was chosen after several tests. Its symmetry divides by a factor 12 the number of cases for a sound ray to propagate from one building to another. Further, the width is the same for all streets and the buildings on both sides of the street are parallel. It is also possible to vary at a certain range the respective values of the size of building facades and the street widths, as long as the horizon remains finite. As illustrated by Fig. 4, the number of ways to connect a building to another one is 108, but only 9 if the symmetry property of the buildings is taken into account. The coordinates on the perimeter of a building and the velocity direction of a sound ray can then be obtained very quickly using 9 continuous linear functions. For each new collision, the impact location increases by an integer value relating to the relative position of the hit building, plus a small value depending on the coordinates on the perimeter.

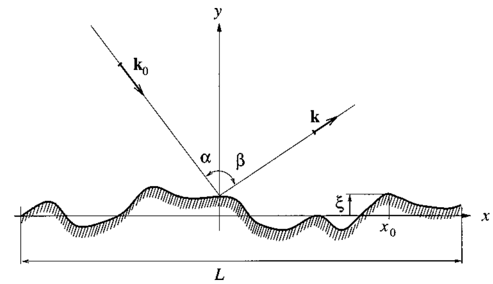

FIG. 5. Wave scattering from a rough surface. $\xi(x_0)$ is the height of the irregularity at position x_0 (according to a Gaussian distribution in our case). $\mathbf{k}_0$ and $\mathbf{k}$ are, respectively, the incident and the scattered wave vectors.

B. Sound scattering by building facades

The organization of the streets in the urban space defines the propagation domain for the noise. In order to achieve the definition of the proposed model, and because it is a major factor affecting urban noise propagation, the reflection laws of the acoustical waves on buildings are expressed in this section.

When the dimensions of the facade irregularities are shorter than the wavelength, walls are like mirrors and a specular reflection occurs: it is the way used by most of the urban noise propagation models. When irregularities are large compared to the wavelength, sound is scattered in all directions with various amplitudes. Unfortunately, only very few research works have been done on the modeling of sound scattering by building facades. First, Lyon *et al.* [24] and Davies [12] assumed that, for each reflection, a fraction of the incident sound power is reflected specularly while another fraction is scattered uniformly in all directions. Derived from the works investigated by Chien and Carroll about the reflection of sound above a rough absorbent plane [42], Makarewicz and co-workers [21,43], Wu and Kittinger [17], and Heutschi [15] used a random scattering according to Lambert's law to model sound diffraction by buildings.

The choice made in this paper is to reflect waves according to several laws: (i) a uniform law—the direction of the reflected ray is chosen randomly according to a uniform law (independent of the direction of the incident ray), (ii) a Lambert's law—the reflected ray is chosen randomly according to Lambert's law (independent of the direction of the incident ray), and (iii) a semidiffuse law. In this latter case, the ray is reflected around the specular reflection direction (meaning that the random law depends on the incident ray). Such a law was recently proposed and discussed in room acoustics by Hidaka [44]. The theory is succinctly described here. Considering an incident plane wave (Fig. 5)

$$\exp(i\mathbf{k}_0\cdot\mathbf{r}) \tag{1}$$

reflected by an one-dimensional irregular surface with finite size L and height $\xi(x)$, and using Green's theorem and Kirchhoff's boundary condition, the scattering coefficient ρ may be expressed by [45–47]

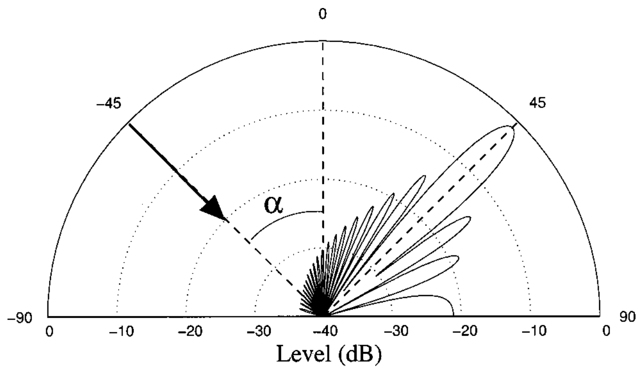

FIG. 6. Sound scattering by a rough surface. $\alpha=\pi/4$. $N=100$ Hz, $L=20$ m, and $\sigma=0.2$ m.

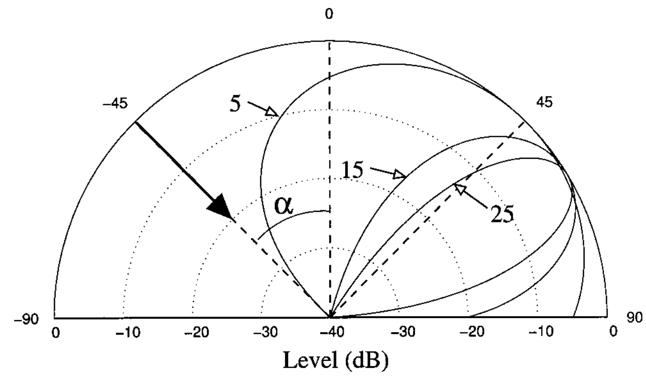

FIG. 7. Sound scattering by a rough surface. $\alpha=\pi/4$. $N=1000$ Hz, $L=50$ m, and $L_c/\sigma=5$,15, and 25.

$$\rho=\frac{H}{L}\int_L \exp(i\mathbf{v}\cdot\mathbf{r})dx, \tag{2}$$

with

$$\mathbf{v}=(v_x,v_y)=k(\sin\alpha-\sin\beta,\cos\alpha+\cos\beta) \tag{3}$$

and

$$H=\frac{1+\cos(\alpha+\beta)}{\cos\alpha+\cos\beta}\sec\alpha, \tag{4}$$

where k and sec are, respectively, the amplitude of the reflected wave number $\mathbf{k}$ and the secant function. The mean scattered power is then proportional to

$$\langle\rho\rho^*\rangle=\langle\rho\rangle\langle\rho^*\rangle+\langle|\rho-\langle\rho\rangle|^2\rangle. \tag{5}$$

If $\xi(x)$ follows a normally distributed random process $N(0,\sigma)$ with a Gaussian spatial-correlation function (defined by its correlation length L_c), Hidaka shows that $\langle\rho\rho^*\rangle$ may be theoretically written on an integral form, but cannot be evaluated. Nevertheless, when $\sigma^2 v_y^2 \ll 1$, the mean scattered power in the β direction may be approximated by [44]

$$\langle\rho\rho^*\rangle=\left\{\left[\frac{\sin(v_x L/2)}{v_x L/2}\right]^2+\bigcirc(\sigma^2 v_y^2)\right\}\exp(-\sigma^2 v_y^2). \tag{6}$$

As shown by Fig. 6, the behavior of the reflected wave is clearly specular for lower frequencies and small facade irregularities.

In the opposite case, when $\sigma^2 v_y^2 \gg 1$, the following equation is obtained [44]:

$$\langle\rho\rho^*\rangle=\sqrt{\pi}\,\frac{H^2}{\sigma v_y}\,\frac{L_c}{L}\exp\left(\frac{-v_x^2 L_c^2}{4\sigma^2 v_y^2}\right). \tag{7}$$

For high frequencies and large irregularities, the behavior is then more diffusing (Fig. 7), but the specular reflection is still dominant. However, when L_c/σ decreases, which means randomness is large and correlation L_c is short, the scattering pattern tends to Lambert's law.

Last, this reflection law allows us to change continuously from a specular reflection to a very diffusing behavior like Lambert's law. At this point, it is important to notice that the perfectly specular law is not used in the simulations because it was proved that, with perfectly reflecting buildings, the behavior is very special [48,49].

III. NUMERICAL SIMULATION

A. Method

For practical simulations, harmonic source is located on the surface of one building called the origin building. The departure ray is chosen randomly with $-\pi/2<\beta<\pi/2$ with a uniform distribution law (β represents the angle between the ray and the normal to the wall). It is convenient to use the curvilinear abscissa s on the perimeter and the coordinates (X_i,Y_i) of the center of the building on which the sound particle is reflected (processing by discrete values since the model is periodic in its definition). As soon as the abscissa s and the departure angle are chosen, one obtains from tedious but elementary geometrical considerations two deterministic recurrence laws T and F (see Appendix), respectively, for the curvilinear abscissa s_{i+1} and the incident angle α_{i+1}, and for the coordinates of the new considered building,

$$(\alpha_{i+1},s_{i+1})=T(\beta_i,s_i) \tag{8}$$

and

$$(\Delta X_i,\Delta Y_i)=(X_{i+1}-X_i,Y_{i+1}-Y_i)=F(\beta_i,s_i). \tag{9}$$

In order to take into account the diffraction process, a random choice for the next departure direction for the reflected ray according to the roughness for the building facades is then realized according to the reflection probability law. So, studying the iterates $(\beta_0,s_0),(\beta_1,s_1),\ldots,(\beta_n,s_n)$, allows us to follow with accuracy the path of a sound particle through the network of buildings. In practice, a rejection method [50] is used to obtain a random reflection angle β_{i+1} verifying the angular probability distribution $P(\alpha_{i+1},\theta)$ depending on the incident angle α_{i+1} and with a maximum for the specular reflection

$$P_{\max}=P(\alpha_{i+1},-\alpha_{i+1}). \tag{10}$$

The idea is to consider a pair of independent and uniformly distributed random variables (θ,γ) satisfying

$$-\pi/2<\theta<\pi/2, \tag{11a}$$

$$0\leqslant\gamma\leqslant P_{\max}. \tag{11b}$$

Then, if the pair of random numbers satisfies the condition

$$\theta \leq P(\alpha_{i+1}, \gamma), \tag{12}$$

$\beta_{i+1} \equiv \theta$. If this condition is not satisfied, the choice is simply rejected. Clearly, by iterating again a number of times this process, a density of random choices around a value θ_0 is obtained, in a proportion $P(\alpha_{i+1}, \theta_0)$, as expected. The advantage of using a recurrence law is that it does not consume computation time, allowing the possibility of ray tracing a great number of sound particles and then obtaining a good expected value estimation. In the simulation, all the dissipative effects such as atmospheric attenuation (depending on humidity and temperature), wall absorption (depending on the nature of facades), and absorption by the open top, are neglected. Although it is conceivable to take into account these processes in the simulation, the choice was done to ignore them for simplicity.

B. The diffusion

The position $\mathbf{R}_j$ of the sound particle j after N displacements in the network of hexagonal buildings is given by

$$\mathbf{R}_j = \sum_{i=1}^{N} \mathbf{r}_i, \tag{13}$$

where $\mathbf{r}_i$ is the displacement between two consecutive collisions i and $i+1$ with buildings (Fig. 4). The time t during which the sound particle suffers N collisions is then proportional to the traveled distance and is given by

$$t = \sum_{i=1}^{N} \frac{|\mathbf{r}_i|}{c}, \tag{14}$$

where c is the velocity of the sound particles. Then, considering a great number M of particles radiated by a source at $t=0$, and averaging over the position of each particle (Fig. 8), allows us to describe statistically the mean-square position $\langle \mathbf{R}^2(t) \rangle$ of the sound particles at the time t:

$$\langle \mathbf{R}^2(t) \rangle = \frac{1}{M} \sum_{j=1}^{M} \mathbf{R}_j^2. \tag{15}$$

As illustrated by Fig. 9, the increase of the value $\langle \mathbf{R}^2(t) \rangle$ is approximately linear with time, except for larger propagation time where the mean-square distance form source seems to reach a plateau. The accuracy of the approximation of $\langle \mathbf{R}^2(t) \rangle$ in terms of a linear function depends on the quality of the estimation of the ensemble average $\langle \quad \rangle$. For the largest values of the time, an average on a large number of initial conditions is needed in order to improve the quality of the estimation of $\langle \mathbf{R}^2(t) \rangle$.

Then, the mean-square distance from source may be approximated by

$$\langle \mathbf{R}^2(t) \rangle = 4Dt, \tag{16}$$

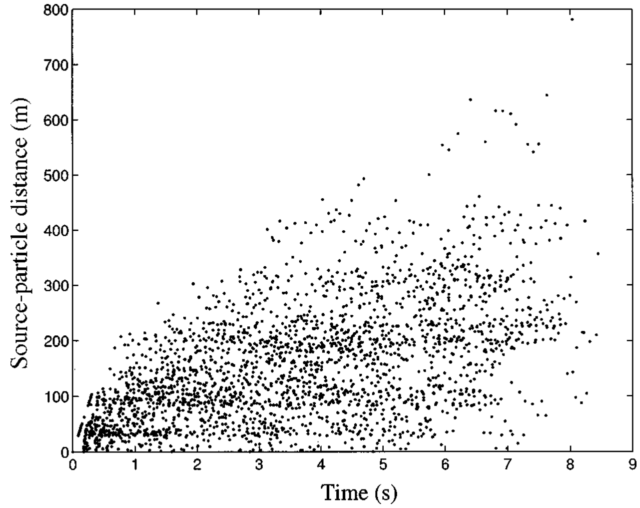

FIG. 8. Propagation of 50 sound particles in the network of hexagonal buildings ($L=113$ m, $d=29.4$ m), each particle suffering 50 collisions with buildings. Each point represents the distance $\mathbf{R}(t)$ between the sound particle and the sound source at each collision as a function of time.

which is a typical result of a process of diffusion [51] and where D is the coefficient of diffusion. In other words, the probability $W(\mathbf{R},t)$ that a sound particle arrives at point $\mathbf{R}$ and at time t is given by

$$W(\mathbf{R},t) = \frac{1}{4\pi Dt} \exp\left[-\frac{\mathbf{R}^2}{4Dt} \right]. \tag{17}$$

This expression is the fundamental solution of the following standard form of the equation of diffusion:

$$\frac{\partial W}{\partial t} - D \Delta W = 0. \tag{18}$$

In a macroscopic view, $W(\mathbf{R},t)$ denotes the concentration of sound particles at position $\mathbf{R}$ and at time t.

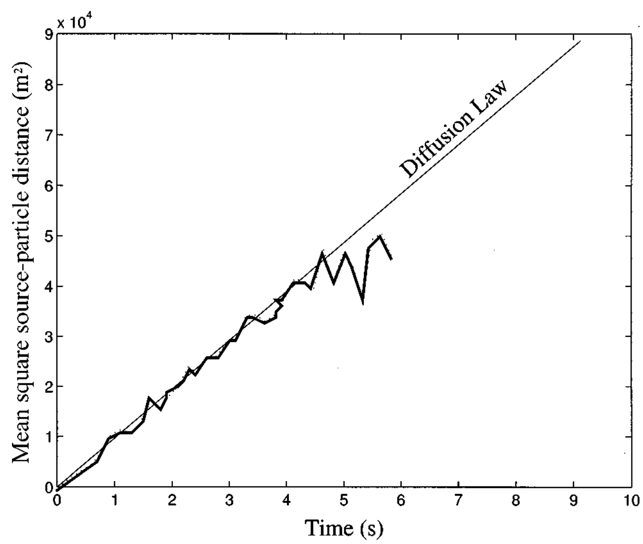

FIG. 9. The thick line and the right line are, respectively, the mean-square distance $\langle \mathbf{R}^2(t) \rangle$ and the approximation by a diffusion law $\langle \mathbf{R}^2(t) \rangle = 4Dt$ with $D=2431$ m^2/s.

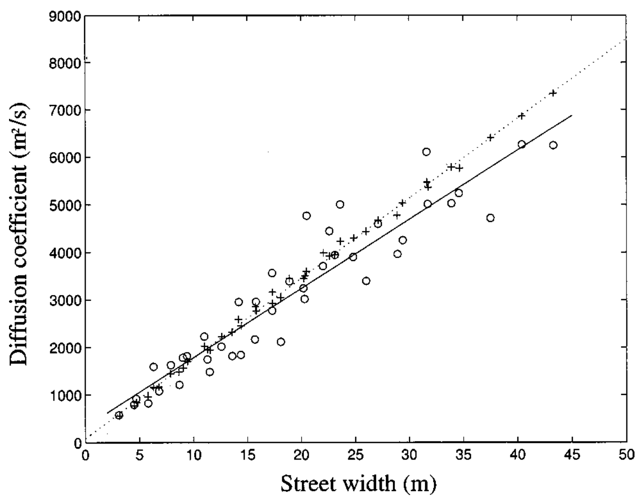

FIG. 10. Diffusion coefficient D as a function of the street width d, for uniform reflection. ○, numerical estimation of D; $+$, diffusion Coefficient D_f according to Eq. (19) and calculated from the values of n and $\langle r^2 \rangle$ obtained by the numerical simulations. The dotted line represents the linear approximation of D_f. The full line is the linear approximation of the coefficient of diffusion D.

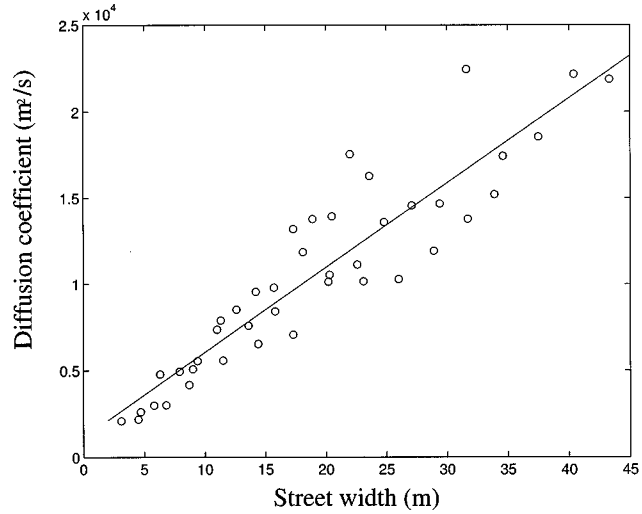

FIG. 12. Diffusion coefficient D as a function of the street width d. ○, numerical estimation for a semidiffuse reflection law with $L_c/\sigma=5$. The full line is the linear approximation of the coefficient of diffusion D.

Simple representations of the evolution of the coefficient of diffusion are given by Figs. 10–13 as a function of the street width and for several reflection laws. At first, it is easy to see that the dispersion of the coefficient of diffusion around its linear approximation increases with the street widths. In order to obtain a better accuracy on the value of D, it would be necessary to consider a greater number of sound particles and collisions, which increases drastically the computation time. Nevertheless, the aim of this paper is not to find an exact value of the coefficient of diffusion, but only to show that the sound propagation can be reduced to a process of diffusion. As illustrated by Figs. 10–13, it is reasonable to think that the increase of D with the street width is approximately linear within an error $\Delta D/D$ less than 30% in the most unfavorable case. In other words, the larger the streets are, the faster the diffusion occurs. Moreover, it is interesting to note that for the uniform reflection law (Fig. 10), the values of D are close to the coefficient of diffusion D_f given by the random-walk theory with a spherical distribution of displacements [51]:

$$D_f=\frac{n\langle \mathbf{r}^2 \rangle}{4}, \tag{19}$$

where n is the number of displacements per unit time and $\langle \mathbf{r}^2 \rangle$ is the mean-square value of displacements, estimated by

$$\langle \mathbf{r}^2 \rangle=\frac{1}{M\times N}\sum_{i=1}^{M\times N}\mathbf{r}_i^2 . \tag{20}$$

As shown by Figs. 10 and 11, the diffusion is lower for the Lambert's diffuse reflection than for the uniform reflec-

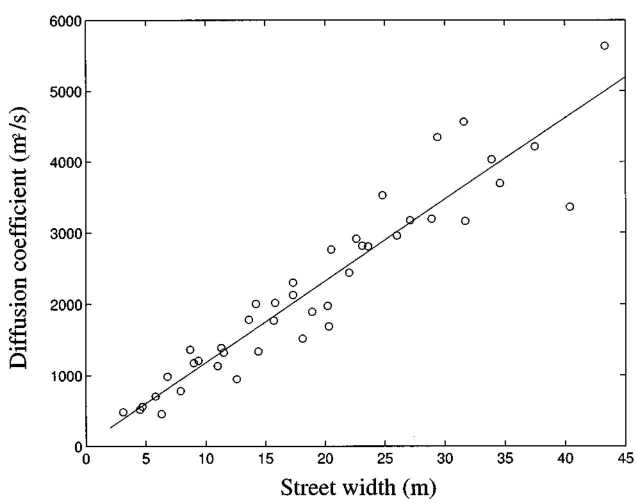

FIG. 11. Diffusion coefficient as a function of the street width d, for diffuse reflection (Lambert's law). ○, numerical estimation of D. The full line is the linear approximation of the coefficient of diffusion D.

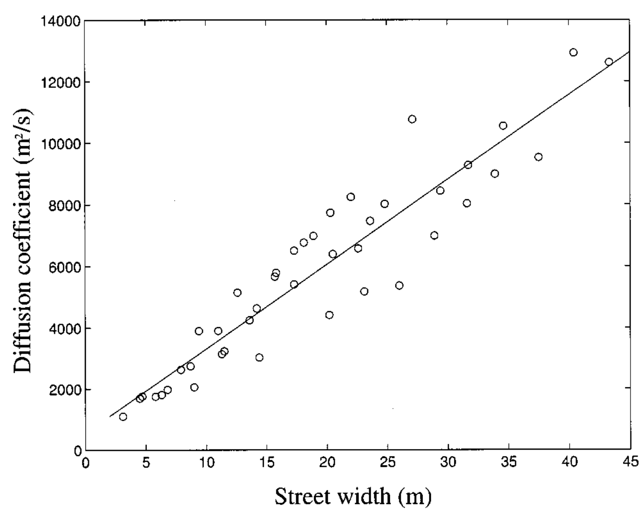

FIG. 13. Diffusion coefficient D as a function of the street width d. ○, numerical estimation for a semidiffuse reflection law with $L_c/\sigma=15$. The full line is the linear approximation of the coefficient of diffusion D.

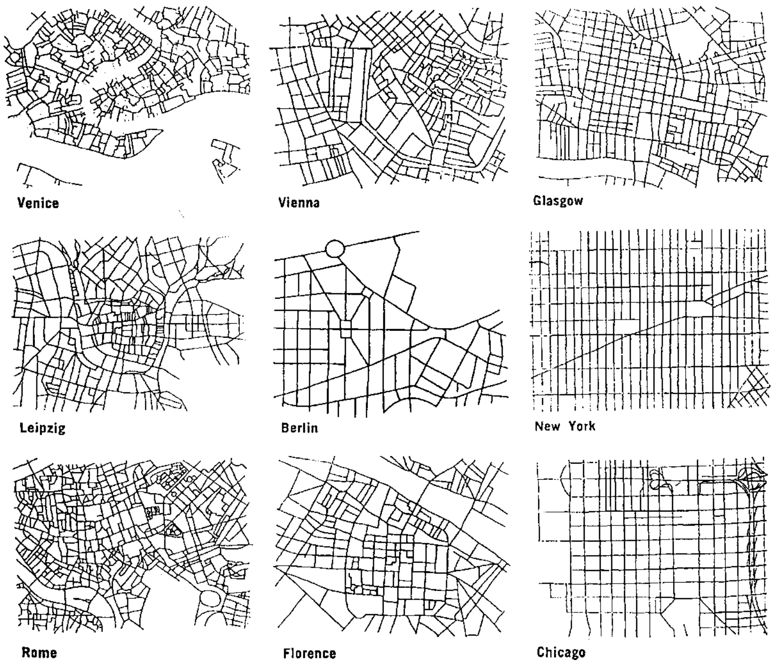

FIG. 14. Typical geometries of urban spaces [52].

tion law. This is in accordance with the fact that the probability for a sound particle to be reflected perpendicular to the wall is maximal for the diffuse reflection law. Then, the sound particle takes more time to spread out in the network of buildings. This idea is validated by Figs. 12 and 13: when reflection tends to be specular, L_c/σ increasing, the diffusion then becomes faster.

IV. SOUND PROPAGATION IN URBAN AREAS

The aim of the present paper has been to show that the background noise in simulated urban areas has a diffusing feature and that the roughness of the building facades is sufficient, without extra hypothesis, to ensure such a behavior. For application to realistic urban cases, it is obvious that this property should remain, because the disposition of streets (Fig. 14) is more similar to a random disposition than a perfectly regular (periodic) one.

To predict practically the noise level from the knowledge of the source distribution, one needs to take into account the dissipative effects. The results of the previous section suggest the adoption of the following equation to describe the spreading out of the energy density $f(\mathbf{R},t)$ of the background noise:

$$\frac{\partial f}{\partial t} - D\Delta f + \eta f = S(\mathbf{R},t), \tag{21}$$

where D and η are, respectively, the diffusion coefficient and the attenuation term (accounting for air attenuation, absorption by walls, and by the open top $\eta > 0$). $S(\mathbf{R},t)$ is the distribution of sound sources. The solution of this equation can be obtained classically by the use of the Green function $G(\mathbf{R},t)$:

$$G(\mathbf{R},t) = \frac{1}{4\pi Dt}\exp\left[-\frac{\mathbf{R}^2}{4\pi Dt} - \eta t\right], \tag{22}$$

which is solution of the equation

$$\frac{\partial G}{\partial t} - D\Delta G + \eta G = \delta(\mathbf{R},t). \tag{23}$$

In fact, the solution $f(\mathbf{R},t)$ is simply the convolution (noted $*$) between $G(\mathbf{R},t)$ and $S(\mathbf{R},t)$:

$$\left(\frac{\partial}{\partial t} - D\Delta + \eta\right)(G*S) = \left[\left(\frac{\partial}{\partial t} - D\Delta + \eta\right)G\right]*S = \delta*S = S. \tag{24}$$

It is useful to remark that in this equation the solution $f(\mathbf{R},t)$ can be applied only inside the streets. This remark makes Eq. (21) somewhat symbolic.

The coefficient of diffusion D, as shown in this paper, depends on the geometrical disposition of the buildings in the built-up area, on the frequency, and on the roughness of the walls. The attenuation coefficient depends on the atmospheric conditions (temperature, humidity, etc.), the frequency, the nature of building facades, and the opened surfaces. The relative importance of these two parameters determines the behavior of the noise, either diffusing or dissipative. Very often, both behaviors occur for the same built-up area but for different frequency ranges.

Because there is an infinity of geometries of urban spaces, it would be unrealistic to find an exact expression for D and σ for each configuration. On the other hand, a statistical study of the urban morphological parameters makes it possible to describe the urban spaces by mean values. Then, the measurement of the coefficient of diffusion in a few typical urban spaces (Fig. 14) could be sufficient to describe statistically most of the other configurations. If this hypothesis is verified, the use of Eq. (21) can be very useful for practical prediction of background noise.

V. CONCLUSION

It was shown that the diffraction and the multiple reflections of the sound waves on building facades imply a diffus-

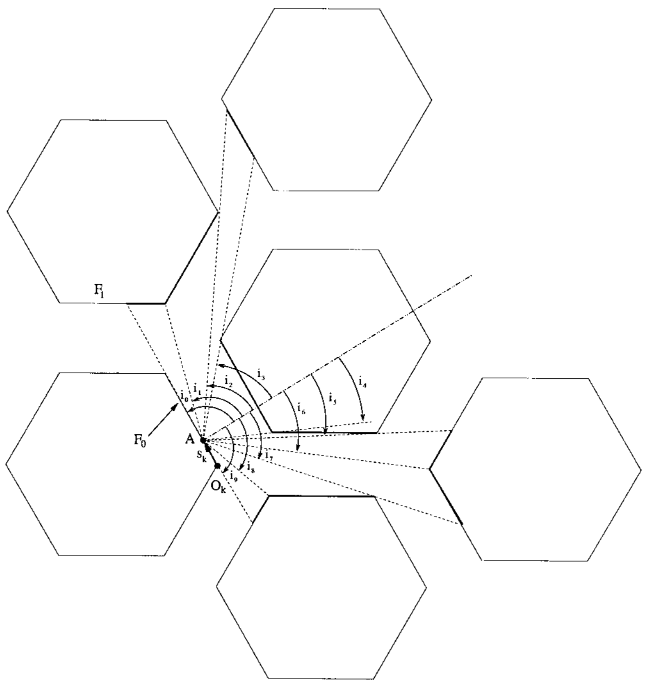

FIG. 15. Propagation paths from the building facade F_0 to the surrounding buildings. Definition of the limit angles.

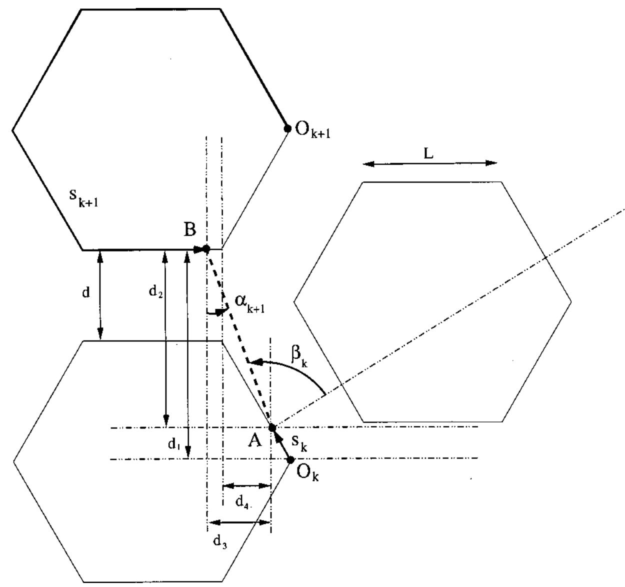

FIG. 16. Definition of the geometry. Propagation of a sound particle from A (defined by its curvilinear abscissa s_k on the incident building) to B (defined by its curvilinear abscissa s_{k+1} on the next building).

ing behavior for the acoustical rays in the case of a periodic model of a city. As a consequence, such a behavior is likely for realistic cities. Then, a modified diffusion equation with two parameters could be used to predict background noise. It is expected that this equation could be implemented numerically for noise prediction.

APPENDIX: DETERMINATION OF THE RECURRENCE LAWS *T* AND *F*

The two recurrence laws T and F have been built by considering both elemental geometrical relations and symmetries of the network of buildings. Here, an example of resolution is proposed to describe the propagation of a sound particle from a building facade to another one.

Let us assume that a sound particle is propagating in the direction β_k from a point A, defined by its curvilinear abscissa s_k on the facade F_0 of a building (X_k, Y_k). Notations are detailed in Figs. 15 and 16.

First, by considering the curvilinear abscissa r_k of A and by taking into account the building size, one can determine the limit angles $i_0, i_1, \ldots, i_9$, which characterize the nine possible propagation paths to the surrounding buildings (see Fig. 15). For example, considering the case Q_1 defined in Fig. 4, the building facade F_1 is reached if β_k is included between i_0 and i_1, defined by

$$i_0 = \frac{\pi}{2}, \tag{A1a}$$

$$i_1 = \frac{5\pi}{6} - \tan^{-1}\frac{2d + (L - s_k)\sqrt{3}}{d - s_k}, \tag{A1b}$$

where d and L are, respectively, the distance between two parallel facades and the length of a building facade. Furthermore, these angles depend also on the building size. From the knowledge of the direction of propagation β_k, it is then easy to determine the building (X_{k+1}, Y_{k+1}), which is reached by the sound particle. In the present example, the position of the building, which is reached by the sound particle, is defined by

$$X_{k+1} = X_k, \tag{A2a}$$

$$Y_{k+1} = Y_k + L\sqrt{3} + d. \tag{A2b}$$

By applying simple geometrical relations, the incident angle of the sound particle on the building facade F_1 leads to (Fig. 16)

$$\alpha_{k+1} = \beta_k - \frac{\pi}{3}. \tag{A3}$$

The curvilinear abscissa s_{k+1} of B on the new building is then

$$s_{k+1} = 5L - d_4 + d_3, \tag{A4}$$

where

$$d_4 = \frac{L + s_k}{2}, \tag{A5a}$$

$$d_3 = d_2 \tan \alpha_{k+1}, \tag{A5b}$$

$$d_2 = d + (L + s_k)\frac{\sqrt{3}}{2}. \tag{A5c}$$

A similar approach may be detailed for the other solutions $Q_2, Q_3, \ldots, Q_9$ defined in Fig. 4.

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